Inspiring Greatness
INDEPENDENCE SCHOOL DISTRICT

Math Virtual Learning

Calculus AB

Integration using u-substitution

May 13, 2020

Calculus AB
Lesson: May 13, 2020

Objective/Learning Target:
Lesson 3 Integrals Review
Students will evaluate integrals using u-substitution.

Warm-Up:

Note: This is a review. For more examples refer back to your notes.

Watch Videos: u-substitution

Read Article: u-substitution

Notes:

Key takeaway #1: u-substitution is really all about *reversing the chain rule*:

- According to the chain rule, the derivative of $w(u(x))$ is $w'(u(x)) \cdot u'(x)$.
- In u-substitution, we take an expression of the form $w'(u(x)) \cdot u'(x)$ and find its antiderivative $w(u(x))$.

Key takeaway #2: u-substitution helps us take a messy expression and simplify it by making the "inner" function the variable.

Examples:

Find the integral $\int (3x+2)^5 dx$.

Solution.

We make the substitution $u = 3x + 2$. Then

$$du = d(3x+2) = 3dx.$$

So the differential dx is given by

$$dx = \frac{du}{3}.$$

Plug all this in the integral:

$$\int (3x+2)^5 dx = \int u^5 \frac{du}{3} = \frac{1}{3}\int u^5 du = \frac{1}{3}\cdot\frac{u^6}{6} + C = \frac{u^6}{18} + C = \frac{(3x+2)^6}{18} + C.$$

Examples:

Find the integral $\int \frac{dx}{\sqrt{1+4x}}$.

Solution.

We can try to use the substitution $u = 1 + 4x$. Hence

$$du = d\left(1+4x\right) = 4dx,$$

so

$$dx = \frac{du}{4}.$$

This yields

$$\int \frac{dx}{\sqrt{1+4x}} = \int \frac{\frac{du}{4}}{\sqrt{u}} = \frac{1}{4}\int \frac{du}{\sqrt{u}} = \frac{1}{4}\int u^{-\frac{1}{2}}du = \frac{1}{4}\cdot\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{1}{4}\cdot 2u^{\frac{1}{2}} + C$$
$$= \frac{u^{\frac{1}{2}}}{2} + C = \frac{\sqrt{u}}{2} + C = \frac{\sqrt{1+4x}}{2} + C.$$

Practice:

1) Evaluate the integral $\int \frac{x dx}{\sqrt{1+x^2}}$.

2) Evaluate the integral $\int x\sqrt{x+1}dx$.

Answer Key:

Once you have completed the problems, check your answers here.

1) *Solution.*

Let $u = 1 + x^2$. Then

$$du = d\left(1 + x^2\right) = 2x dx.$$

We see that

$$x dx = \frac{du}{2}.$$

Hence

$$\int \frac{x dx}{\sqrt{1+x^2}} = \int \frac{\frac{du}{2}}{\sqrt{u}} = \int \frac{du}{2\sqrt{u}} = \sqrt{u} + C = \sqrt{1+x^2} + C.$$

Answer Key:

Once you have completed the problems, check your answers here.

2) *Solution.*

To get rid of the square root, we make the substitution $u = \sqrt{x+1}$. Then

$$u^2 = x + 1, \quad \Rightarrow x = u^2 - 1, \quad \Rightarrow du = 2udu.$$

The integral becomes

$$\int x\sqrt{x+1}dx = \int \left(u^2 - 1\right) u \cdot 2udu = 2\int \left(u^2 - 1\right) u^2 du = 2\int \left(u^4 - u^2\right) du$$

$$= 2\int u^4 du - 2\int u^2 du = 2 \cdot \frac{u^5}{5} - 2 \cdot \frac{u^3}{3} + C = \frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} + C.$$

Additional Practice:

Interactive Practice

More Interactive Practice

Extra Practice with Answers